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LINEAR CONNECTIONS IN THE BUNDLE OF LINEAR FRAMES

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Abstract. Let $L(M)$ be the bundle of all linear frames over M, u an arbitrarily given point of $L(M)$, and $\nabla : \mathfrak{X}(M) \times \mathfrak{X}(M) \to \mathfrak{X}(M)$ a linear connection on M. Then the following results are well known: the horizontal subspace and the connection form at the point u may be written in terms of local coordinates of $u \in L(M)$ and Christoffel's symbols defined by ∇. These results are very fundamental on the study of the theory of connections. In this paper we show that the local expressions of those at the point u do not depend on the choice of a local coordinate system around the point $u \in L(M)$, which is rarely seen. Moreover we give full explanations for the following fact: the covariant derivative on M which is defined by the parallelism on $L(M)$, determined from the connection form above, coincides with ∇.

1. Introduction

Let $L(M)$ be the bundle of all linear frames over a smooth manifold M, u an arbitrarily given point of $L(M)$, and $\nabla : \mathfrak{X}(M) \times \mathfrak{X}(M) \to \mathfrak{X}(M)$ a linear connection on M. Here $\mathfrak{X}(M)$ is the set of all smooth vector fields on M. In this paper, in terms of local coefficient functions Γ^i_{jk} of the linear connection ∇ on M and local coordinates of the point $u \in L(M)$, we express the horizontal subspace $Q_u \subset T_u(L(M))$ and the connection form at the point $u \in L(M)$ (cf. Proposition 2.2 and Theorem 2.3). These results are well known, and are very fundamental in the study on the theory of connections. In this paper we show that the local expressions of the horizontal subspace and the connection form

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at the point u do not depend on the choice of a local coordinate system around the point $u \in L(M)$, which are rarely seen. Moreover, we give full explanations for the following fact: the covariant derivative on M which is defined by the parallelism on M, determined from the connection form above on $L(M)$, coincides with ∇ (cf. Proposition 2.5).

2. Horizontal subspaces in the bundle of linear frames over a smooth manifold

2.1. Connections in a principal fiber bundle. Let (P, M, G, π) be a principal fiber bundle over a manifold M with group G. For each $u \in P$, let $T_u(P)$ be the tangent space of P at u and G_u the subspace of $T_u(P)$ consisting of vectors tangent to the fiber through u. A *connection* in (P, M, G, π) is an assignment of a subspace Q_u of $T_u(P)$ to each $u \in P$ such that

(2.1) $T_u(P) = G_u + Q_u$ (direct sum);

(2.2) $Q_{ug} = R_{g_\star} Q_u$ for every $u \in P$ and $g \in G$, where R_g is the transformation of P induced by $g \in G$, $R_g u = ug$;
(2.3) Q_u depends differentiably on u.

Given a connection Q in (P, M, G, π), we define a 1-form ω on P with values in the Lie algebra $\mathfrak{g}$ of G as follows. For each $W \in T_u P$, we define $\omega(W)$ to be the unique $x \in \mathfrak{g}$ such that $(x^\star)_u$ is equal to the vertical component of W. Here $x^\star$ is the fundamental vector field corresponding to $x \in \mathfrak{g}$ which is defined on P (cf. [1, 2]). The form ω is called the *connection form* of the given connection Q (cf. [2, 3, 4]).

2.2. The Christoffel's symbols of a linear connection on M. Let $\nabla : \mathfrak{X}(M) \times \mathfrak{X}(M) \to \mathfrak{X}(M)$ be a linear connection on M. Taking local coordinate systems $(x^1, ..., x^n), (y^1, ..., y^n)$ on neighborhoods U, V being contained in M respectively, then we may write

$$\begin{aligned} \nabla_{\frac{\partial}{\partial x^j}} \frac{\partial}{\partial x^i} &= \sum_{k=1}^{n} \Gamma_{ji}^k \frac{\partial}{\partial x^k}, \\ \nabla_{\frac{\partial}{\partial y^j}} \frac{\partial}{\partial y^i} &= \sum_{k=1}^{n} \tilde{\Gamma}_{ji}^k \frac{\partial}{\partial y^k} \end{aligned} \tag{2.4}$$

on U, V respectively. Here Γ_{ji}^k (resp. $\tilde{\Gamma}_{ji}^k$) are called the *Christoffel's symbols* for the linear connection ∇ on M relative to the local coordinate system $(x^1, ..., x^n)$ (resp. $(y^1, ..., y^n)$) on neighborhoods U (resp. V) on

M. In the intersection of the two coordinate neighborhoods U and V, we have

$$\tilde{\Gamma}^k_{ji} = \textstyle\sum_{l,s,h} \frac{\partial y^k}{\partial x^l} \frac{\partial x^s}{\partial y^j} \frac{\partial x^h}{\partial y^i} \Gamma^l_{sh} + \sum_l \frac{\partial^2 x^l}{\partial y^j \partial y^i} \frac{\partial y^k}{\partial x^l}. \tag{2.5}$$

2.3. Horizontal subspaces and connection form. Here and from now on in this paper, we shall denote the bundle $L(M)$ of linear frames on M by P and the general linear group $GL(n, \mathbb{R})$, $n = \dim M$, by G. Let $\nabla : \mathfrak{X}(M) \times \mathfrak{X}(M) \to \mathfrak{X}(M)$ be a linear connection on M. Let Γ^k_{ji} and $\tilde{\Gamma}^k_{ji}$ the Christoffel's symbols with respect to local coordinate systems $x^1, ..., x^n$, $y^1, ..., y^n$ which are defined on neighborhoods U, V, respectively. For $w \in \pi^{-1}(U \cap V)$ we may write

$$\begin{aligned} w =& (\frac{\partial}{\partial x^1}, ..., \frac{\partial}{\partial x^n})_{\pi(w)} ({a^i}_j(\pi(w)))_{i,j} \\ =& (\frac{\partial}{\partial y^1}, ..., \frac{\partial}{\partial y^n})_{\pi(w)} ({b^i}_j(\pi(w)))_{i,j}. \end{aligned} \tag{2.6}$$

So we obtain two local coordinate systems on $\pi^{-1}(U \cap V)$

$$(x^1, ..., x^n, {a^1}_1, ..., {a^n}_n) \text{ and } (y^1, ..., y^n, {b^1}_1, ..., {b^n}_n). \tag{2.7}$$

Let p be an arbitrarily given point which belongs to U, and let $X = \sum_i \xi^i (\partial/\partial x^i)_p = \sum_i \eta^i (\partial/\partial y^i)_p$, $(\xi^i, \eta^i \in \mathbb{R})$, be an arbitrarily given tangent vector which belongs to $T_p(M)$. Then

$$\eta^i = \textstyle\sum_j \xi^j (\partial y^i/\partial x^j)_p \in \mathbb{R}. \tag{2.8}$$

First of all, we obtain the horizontal lift of X at $u \in P$, $(\pi(u) = p)$. Let $c_t, -\epsilon < t < \epsilon$, be an integral curve of X which satisfies the following conditions:

$$\begin{aligned} & c_0 = p = \pi(u), \\ & (x^1(c_t), ..., x^n(c_t)) = (\xi^1 t, ..., \xi^n t) \quad (-\epsilon < t < \epsilon). \end{aligned} \tag{2.9}$$

Let u_t, $-\epsilon < t < \epsilon$, be the horizontal lift of c_t satisfying the initial condition $u_0 = u$. We denote u_t, $-\epsilon < t < \epsilon$, by $(X_1(t), ..., X_n(t))$. Then, $(X_1(0), ..., X_n(0)) = u_0 = u$. Moreover we may write for each $t \in (-\epsilon, \epsilon)$

$$\begin{aligned} u_t =& (X_1(t), ..., X_n(t)) \\ =& (\frac{\partial}{\partial x^1}, ..., \frac{\partial}{\partial x^n})_{c_t} ({a^i}_j(u_t))_{i,j} \\ =& (\frac{\partial}{\partial y^1}, ..., \frac{\partial}{\partial y^n})_{c_t} ({b^i}_j(u_t))_{i,j}. \end{aligned} \tag{2.10}$$

From (2.6) we get on $\pi^{-1}(U \cap V)$

$$b^i{}_j = \textstyle\sum_l \frac{\partial y^i}{\partial x^l} a^l{}_j, \; a^i{}_j = \sum_l \frac{\partial x^i}{\partial y^l} b^l{}_j. \tag{2.11}$$

In order to obtain the main results, we recall the lemma below

LEMMA 2.1 (2). . *Let $v_t = (Y_1(t), ..., Y_n(t))$, $-\epsilon < t < \epsilon$, be a smooth curve in $L(M)$ with $\pi(v_t) = c(t)$. Then the following statements are equivalent*:

(i) each $Y_i(t)$, $-\epsilon < t < \epsilon$, is parallel along the curve $c(t)$;

(ii) v_t, $-\epsilon < t < \epsilon$, is the horizontal lift of the curve $c(t)$ passing through the point v_0.

Since u_t, $-\epsilon < t < \epsilon$, is the horizontal curve of the curve c_t with $u_0 = u$, we get from (2.4), (2.9) and Lemma 2.1

$$\begin{aligned} \nabla_{dc_t/dt} X_j(t) &= \sum_i \nabla_{dc_t/dt} a^i{}_j(u_t)(\partial/\partial x^i)_{c_t} \\ &= \sum_i \{da^i{}_j(u_t)/dt + \sum_{k,l} \xi^l a^k{}_j(u_t)\Gamma^i_{lk}\}(\partial/\partial x^i)_{c_t} = 0. \end{aligned} \tag{2.12}$$

So the horizontal lift $X^\star \in T_u(P)$ of the vector $X = \sum_i \xi^i(\partial/\partial x^i)_p$ may be written as

$$X^\star = \textstyle\sum_i \xi^i(\partial/\partial x^i)_u - \sum_{i,l} \xi^i a^l{}_j(u)\Gamma^k_{il}(p)(\partial/\partial a^k{}_j)_u. \tag{2.13}$$

By virtue of (2.6) and (2.7), we get on $\pi^{-1}(U \cap V)$

$$\begin{aligned} &\frac{\partial}{\partial x^k} = \sum_i \frac{\partial y^i}{\partial x^k}\frac{\partial}{\partial y^i} + \sum_{h,i,l} a^h{}_l \frac{\partial^2 y^i}{\partial x^k \partial x^h}\frac{\partial}{\partial b^i{}_l}, \qquad \frac{\partial}{\partial a^s{}_j} = \sum_i \frac{\partial y^i}{\partial x^s}\frac{\partial}{\partial b^i{}_j}, \\ &dx^i = \sum_j \frac{\partial x^i}{\partial y^j} dy^j, \qquad da^i{}_j = \sum_{k,l} \frac{\partial^2 x^i}{\partial y^k \partial y^l} b^l{}_j dy^k + \sum_l \frac{\partial x^i}{\partial y^l} db^l{}_j, \\ &\frac{\partial}{\partial y^k} = \sum_i \frac{\partial x^i}{\partial y^k}\frac{\partial}{\partial x^i} + \sum_{h,i,l} b^h{}_l \frac{\partial^2 x^i}{\partial y^k \partial y^h}\frac{\partial}{\partial a^i{}_l}, \qquad \frac{\partial}{\partial b^s{}_j} = \sum_i \frac{\partial x^i}{\partial y^s}\frac{\partial}{\partial a^i{}_j}, \\ &dy^i = \sum_j \frac{\partial y^i}{\partial x^j} dx^j, \qquad db^i{}_j = \sum_{k,l} \frac{\partial^2 y^i}{\partial x^k \partial x^l} a^l{}_j dx^k + \sum_l \frac{\partial y^i}{\partial x^l} da^l{}_j. \end{aligned} \tag{2.14}$$

From (2.5), (2.8), (2.11), (2.13) and (2.14), we obtain

$$\begin{aligned} X^\star &= \sum_i \xi^i(\partial/\partial x^i)_u - \sum_{i,l} \xi^i a^l{}_j(u)\Gamma^k_{il}(p)(\partial/\partial a^k{}_j)_u \\ &= \sum_i \eta^i(\partial/\partial y^i)_u - \sum_{i,l} \eta^i b^l{}_j(u)\tilde{\Gamma}^k_{il}(p)(\partial/\partial b^k{}_j)_u. \end{aligned} \tag{2.15}$$

This shows that the horizontal subspace does not depend on the choice of the local coordinate system around the point $u \in L(M)$. Thus we have

PROPOSITION 2.2. *Let $u \in L(M)$ be an arbitrarily given point, $(x^1, ..., x^n)$ a coordinate system on a neighborhood $U \subset M$ which contains $\pi(u) = p$, and $(x^1, ..., x^n, a^1{}_1, ..., a^n{}_n)$ the coordinate system on $\pi^{-1}(U) \subset L(M)$. Let Γ^i_{jk} be the Christoffel's symbols on U which are defined by a linear connection $\nabla : \mathfrak{X}(M) \times \mathfrak{X}(M) \to \mathfrak{X}(M)$. Then the horizontal subspace Q_u of $T_uL(M)$ at the point u is given as follows:*

$$Q_u = \{\sum_i \xi^i(\partial/\partial x^i)_u - \sum_{i,l} \xi^i a^l{}_j(u)\Gamma^k_{il}(p)(\frac{\partial}{\partial a^k{}_j})_u \mid each\ \xi^i \in \mathbb{R}\}.$$

Let $E_k{}^j$ denote a square matrix of order n with the (k, j)-entry being 1, and all the other entries being 0. Putting $(a^i{}_j)_{i,j} =: A$, $(b^i{}_j)_{i,j} =: B$, $A^{-1} =: C = (c^i{}_j)_{i,j}$ and $B^{-1} =: D = (d^i{}_j)_{i,j}$, then we have from (2.6)

$$a^i{}_j = \textstyle\sum_l \frac{\partial x^i}{\partial y^l} b^l{}_j,\ c^i{}_j = \sum_l d^i{}_l \frac{\partial y^l}{\partial x^j} \tag{2.16}$$

on $\pi^{-1}(U \cap V)$. Putting

$$\omega^k{}_j := \sum_m c^k{}_m(da^m{}_j + \sum_{i,l} \Gamma^m_{il} a^l{}_j dx^i)$$

on $\pi^{-1}(U)$, we obtain from (2.5), (2.14) and (2.16)

$$\begin{aligned} \omega^k{}_j &= \sum_m c^k{}_m(da^m{}_j + \sum_{i,l} \Gamma^m_{il} a^l{}_j dx^i) \\ &= \sum_m d^k{}_m(db^m{}_j + \sum_{i,l} \tilde{\Gamma}^m_{il} b^l{}_j dy^i) \end{aligned} \tag{2.17}$$

on $\pi^{-1}(U \cap V)$. This shows that the definition of $\omega^k{}_j$ is independent of the choice of the local coordinate system.

Putting $\omega := \sum_{k,j} \omega^k{}_j E_k{}^j$ on $\pi^{-1}(U) \subset P$, we get $R_g{}^\star\omega = \mathrm{Ad}(g^{-1})\omega$ $(g \in GL_n(\mathbb{R}))$ since

$$(R_g{}^\star da^m{}_j)(x^\star{}_u) = \lim_{t\to 0}\{a^m{}_j(ug(g^{-1}\exp(tx)g)) - a^m{}_j(ug)\}/t,$$

$(u \in \pi^{-1}(U), x \in \mathfrak{gl}_n(\mathbb{R}))$. And then, from the definition of ω we obtain $\omega(x^\star{}_u) = x$. Here $x^\star$ is the fundamental vector field on P which is corresponding to $x \in \mathfrak{gl}_n(\mathbb{R})$. These shows that ω is a connection form on P. Moreover $\omega_u(Y) = 0$ for each $Y \in Q_u$, where Q_u is the horizontal subspace of $T_u(P)$ which is appeared in Proposition 2.2. Thus, from Christoffel's symbols we can reconstruct the connection form ω. Summing up we get

THEOREM 2.3. *Using the local coordinate system* $(x^1, ..., x^n, a^1{}_1, ..., a^n{}_n)$ *on* $\pi^{-1}(U)$, *and the Christoffel's symbols* Γ^i_{jk} *with respect to the local coordinate system* $x^1, ..., x^n$ *on* U *and the linear connection* $\nabla : \mathfrak{X}(M) \times \mathfrak{X}(M) \to \mathfrak{X}(M)$, *we may write the connection form* ω *on* $\pi^{-1}(U) \subset L(M)$ *as follows :*

$$\omega = \sum_{j,k,m} c^k{}_m (da^m{}_j + \sum_{i,l} \Gamma^m_{il} a^l{}_j dx^i) E_k{}^j. \tag{2.18}$$

2.4. Covariant derivatives. First of all we introduce the well known

LEMMA 2.4. *Let* $C^\infty(P, \mathbb{R}^n)$ *be the subspace of all smooth maps* f *of* P *into* $\mathbb{R}^n$ *which satisfy* $f(ug) = g^{-1} f(u)$ $(u \in P, g \in GL_n(\mathbb{R}))$. *Then* $\mathfrak{X}(M)$ *and* $C^\infty(P, \mathbb{R}^n)$ *are identified as follows :*
For each $X \in \mathfrak{X}(M)$, $\quad f(u) := u^{-1}(X_{\pi(u)}) \quad (u \in P)$.
Conversely, for each $f \in C^\infty(P, \mathbb{R}^n)$, $\quad X_{\pi(u)} := u(f(u)) \quad (u \in P)$.

The *canonical form* θ of $P(M, G, \pi)$ is the $\mathbb{R}^n$-valued 1-form on P defined by

$$\theta(X) = u^{-1}(\pi_\star(X)) \qquad \text{for } X \in T_u(P). \tag{2.19}$$

Let $Z \in \mathfrak{X}(M)$ and let $\tau = c(t)$, $-\epsilon < t < \epsilon$, be a curve in M. Then, for each fixed t, the *covariant derivative* $\nabla_{dc(t)/dt} Z$ of Z in the direction of $dc(t)/dt \in T_{c(t)}(M)$ is defined by

$$\nabla_{dc(t)/dt} Z = \lim_{h \to 0} \frac{\{\tau_t^{t+h}(Z_{c(t+h)}) - Z_{c(t)}\}}{h}, \tag{2.20}$$

where $\tau_t^{t+h} : T_{c(t+h)}M \to T_{c(t)}M$ denotes the parallel displacement of $T_{c(t+h)}M$ along τ from $c(t+h)$ to $c(t)$. We take a horizontal lift $\tau^\star = v(t)$, $-\epsilon < t < \epsilon$, of $\tau = c(t)$. Then $\tau_t^{t+h}(Z_{c(t+h)})$ in (2.20) becomes

$$\tau_t^{t+h}(Z_{c(t+h)}) = (v(t) \circ v(t+h)^{-1})(Z_{c(t+h)}). \tag{2.21}$$

From (2.20) and (2.21), we have

$$\nabla_{dc(t)/dt} Z = v(t)(\lim_{h \to 0} \frac{v(t+h)^{-1}(Z_{c(t+h)}) - v(t)^{-1}(Z_{c(t)})}{h}). \tag{2.22}$$

By the help of Lemma 2.4 we can associate with Z an $\mathbb{R}^n$-valued function f on P as follows:

$$f(u) = (\theta(Z^\star))(u) = u^{-1}(Z_{\pi(u)}) \qquad (u \in P).$$

Here $Z^\star \in \mathfrak{X}(P)$ is the horizontal lift of $Z \in \mathfrak{X}(M)$.

We put $dc(t)/dt|_{t=0} =: W \in T_p(M)$. Let $W^\star \in T_v(P)$, $(\pi(v) = p,\ v_0 =: v)$, be the horizontal lift of W at v. Since $\theta(Z^\star)$ coincides with f in the sense of Lemma 2.4, by virtue of (2.20) and (2.22) we get

$$\nabla_W Z = v(W^\star f). \tag{2.23}$$

Now, let's apply (2.23) to the following case:

$$W =: (\frac{\partial}{\partial x^j})_p \quad \text{and} \quad Z =: \frac{\partial}{\partial x^i} \qquad \text{on } U \subset M.$$

Let $\sigma : U \to P$ be the cross section of P over U which assigns to each $q \in U \subset M$ the linear frame

$$((\frac{\partial}{\partial x^1})_q, ..., (\frac{\partial}{\partial x^n})_q).$$

We put $f := \sum_k c^k{}_i e_k$ on $\pi^{-1}(U) \subset P$, where $(c^i{}_j)_{i,j}$ is the inverse matrix of the matrix $(a^i{}_j)_{i,j}$ and e_k is the column vector which is belonging to $\mathbb{R}^n$ with the k-th entry being 1, and all the other entries being 0. Then the function f on $\pi^{-1}(U) \subset P$ is the function corresponding to $\frac{\partial}{\partial x^i}$. In fact, since $a^i{}_j(ug) = \sum_l a^i{}_l(u) g^l{}_j \quad (g = (g^i{}_j)_{i,j} \in G, u \in \pi^{-1}(U))$ and $\sum_l c^i{}_l a^l{}_j = \delta^i{}_j$ on $\pi^{-1}(U \cap V)$, $c^i{}_j(ug) = \sum_l h^i{}_l c^l{}_j(u) \quad (g^{-1} =: h = (h^i{}_j)_{i,j} \in G)$. So $f(ug) = g^{-1} f(u) \quad (u \in \pi^{-1}(U),\ g \in G)$. And then, $f(\sigma(q)) = \sum_k c^k{}_i(\sigma(q)) e_k = e_i \quad (q \in U)$, since $a^k{}_i(\sigma(q)) = \delta^k{}_i \quad (q \in U)$. Evidently $\sigma(q)(f(\sigma(q))) = (\frac{\partial}{\partial x^i})_q \quad (q \in U \subset M)$.

From Proposition 2.2, it follows that, in terms of the coordinate system $(x^i, a^j{}_k)$ on $\pi^{-1}(U)$, the horizontal lift $W^\star$ of $(\frac{\partial}{\partial x^j})_p \quad (\pi(v) = p)$ is given by

$$W^\star = (\tfrac{\partial}{\partial x^j})_v - \textstyle\sum_{i,k,l} \Gamma^i_{jk}(p) a^k{}_l(v) (\tfrac{\partial}{\partial a^i{}_l})_v. \tag{2.24}$$

Since $W^\star(\sum_k a^m{}_k c^k{}_i) = 0$ on $\pi^{-1}(U \cap V)$, $\sum_k \{W^\star(a^m{}_k) c^k{}_i + a^m{}_k W^\star(c^k{}_i)\} = 0$. By virtue of this fact, $f := \sum_k c^k{}_i e_k$ and (2.24), we obtain

$$W^\star(f) = \textstyle\sum_{k,l,m} c^k{}_m(v) \Gamma^m_{ji}(p) e_k. \tag{2.25}$$

Putting $\sigma(p) =: v$, then we get from (2.25)

$$c^k{}_m(\sigma(p)) = \delta^k{}_m, \ \text{and} \quad \sigma(p)(W^\star(f)) = \textstyle\sum_k \Gamma^k_{ji}(p) (\tfrac{\partial}{\partial x^k})_p. \tag{2.26}$$

Thus, from (2.23) and (2.26), we obtain

$$(\nabla_{\frac{\partial}{\partial x^j}} \tfrac{\partial}{\partial x^i})_p = \textstyle\sum_k \Gamma^k_{ji}(p) (\tfrac{\partial}{\partial x^k})_p. \tag{2.27}$$

Summing up, in the sense of (2.20) we obtain by virtue of (2.23)

Proposition 2.5. *Assume the connection form* $\omega = \sum_{i,j} \omega^i{}_j E_i^j$ *with respect to the local coordinate system* $(x^1, ..., x^n, a^1{}_1, ..., a^n{}_n)$ *on* $\pi^{-1}(U)$ *is given by*

$$\omega^i{}_j = \sum_k c^i{}_k (da^k{}_j + \sum_{l,m} \Gamma^k_{ml} a^l{}_j dx^m).$$

Then, on $U \subset M$

$$\nabla_{\frac{\partial}{\partial x^j}} \frac{\partial}{\partial x^i} = \sum_k \Gamma^k_{ji} \frac{\partial}{\partial x^k}.$$

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